Development of an Arbitrary Precision Mathematical Function Table as an Educational Material in the Web Era

Tomonori Kouya
Shizuoka Institute of Science and Technology, Shizuoka 437-8555, Japan
tkouya@cs.sist.ac.jp

Abstract

We have developed "MPFRgraph", which is an arbitrary precision mathematical function table on the Web. The reason for developing this application are as follows: 1) to provide an appropriate function table as required by the users in the Web era, 2) to upgrade "Try MPFR!" service that demonstrates the functions of MPFR/GMP on Web, and 3) to provide one of educational materials to learn some technical elements used in Web service as a whole. In this paper, we describe these purposes and construction of MPFRgraph.
Keywords: multiple precision arithmetic, numerical computation, Web programming, educational material

1 Introduction

In this paper, we describe "MPFRgraph", which is an arbitrary precision mathematical function table and graph on the Web using the GNU MPFR[8] and the GNU MP (GMP) libraries[1].

We have developed MPFRgraph for the following three reasons:

1) to provide a novel mathematical function table suited to the Web era

2) to upgrade "Try MPFR!", which demonstrates the functions of MPFR/GMP libraries

3) to provide educational materials on Web programming to students of universities in Japan

First, we explain reason 1). In 2010, the "NIST Handbook of Mathematical Functions"[7] was published as an update to the globally acknowledged book edited by Abramowitz and Stegun (A & S for short)[6]. The contents of NIST Handbook of Mathematical Functions have been made available as "Digital Library"[5] on the Web since 2003 and before its publication. The recent NIST Handbook describes the mathematical properties of functions. The graphs are equally important as in the book edited by A & S; however, function tables (printed values of functions) have been replaced with links to software routines that calculate the values of the functions.

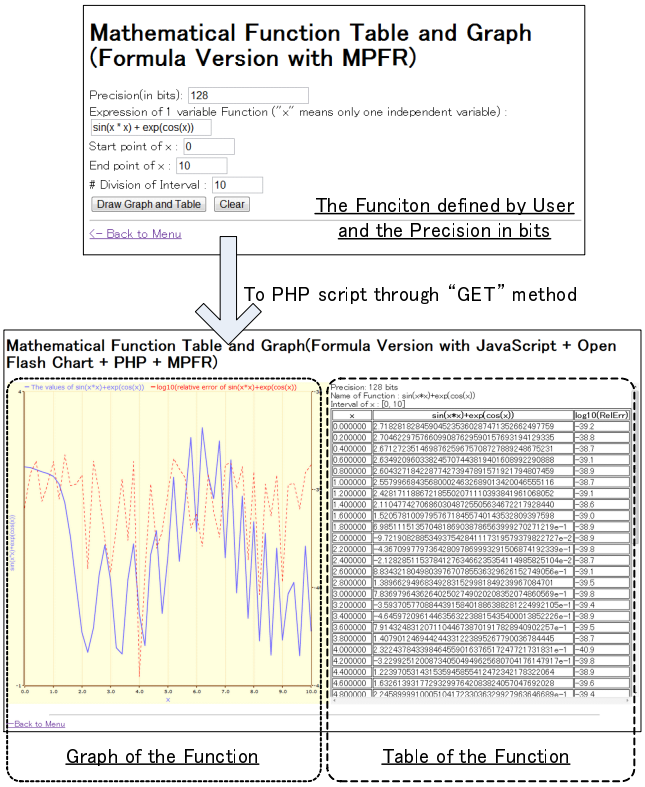


Figure 1: MPFRgraph: Arbitrary precision mathematical function table on Web

The current era allows the calculation of many standard functions through the use of existing reliable software. The newly developed "Mathematical Function Table" helps users to directly obtain the values of functions using such existing software. For instance, $\sqrt{\text{keis}\alpha\text{n}}$ [9] developed by Casio Computer Co., Ltd. and Wolfram|Alpha [4] developed by Wolfram Research Inc., which developed Mathematica, have been freely available on the Web. $\sqrt{\text{keis}\alpha\text{n}}$ supports multiple precision arithmetic up to 50 decimal digits and can plot graphs of functions. Besides Mathematica, Wolfram|Alpha supports arbitrary precision arithmetic, plotting of graphs of functions, and display of the mathematical properties of those functions. Currently, Wolfram|Alpha can be called the greatest mathematical function table on the Web.

MPFRgraph (Figure 1) that has been developed by us, supports arbitrary precision arithmetic and can plot graphs of functions. Furthermore, it can obtain relative errors in the values of functions.

Second, we explain reason 2). We have developed and maintained "Try MPFR!" to demonstrate the availability of MPFR/GMP, which is an arbitrary precision floating-point library since 2004. Try MPFR! can calculate the constant value of an user-defined mathematical expression and functions with user-defined precision (in bits). However, the interface is poorly designed (Figure 2). More functionality is required in the upgraded version of this online application.

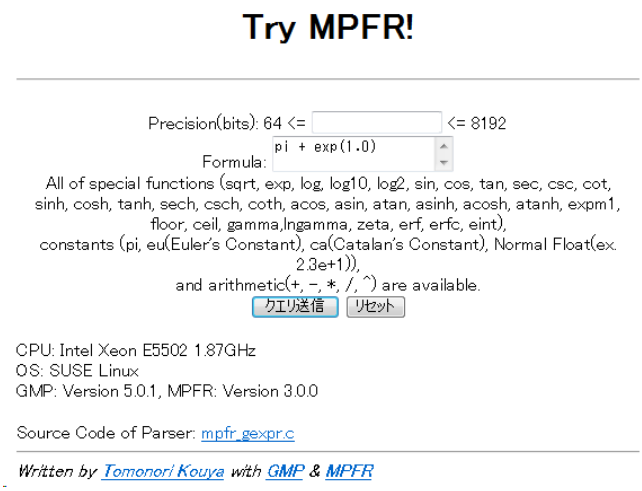


Figure 2: Currently "Try MPFR!" interface

Therefore, we develop the additional faculty based on current Try MPFR!; it can manipulate any user-defined one-variable functions expressed as algebraic combination of some elementary and special functions on any user-defined interval.

Finally, we explain reason 3). Current Web sites are developed using several open-source technologies such as HTML, JavaScript, Flash, and PHP. High-level free tools and libraries such as PHPlot[2] and Open Flash Chart[3] are available on the Internet. MPFRgraph can be considered to be educational material, because it involves such open and free technologies and libraries which our students can learn to build high-level Web sites.

In the rest of this paper, we introduce the MPFR/GMP library and the current Try MPFR!. We later explain the implementation and its execution flow, and show educational materials of Web and native application development to build MPFRgraph. Finally, we conclude this paper and describe future studies.

2 Try MPFR! and MPFR/GMP

GNU MPFR[8] is an excellent arbitrary precision floating-point library based on the multiple precision natural number arithmetic (mpn) that composes a well-tuned kernel of GNU MP library[1]. MPFR must be combined with GMP and is hence denoted as "MPFR/GMP" in this paper.

GMP has a more well-tuned mpn kernel than other arbitrary precision arithmetic libraries; it can be used to design various CPU architectures. GMP supports high-performance arbitrary precision integers, rational numbers and floating-point arithmetic. The floating-point arithmetic in GMP is incompatible with the IEEE754 standard for floating-point arithmetic and does not have elementary and special functions except square root, which is frequently used in scientific computation. However, MPFR/GMP has supported these important functions that can return correctly rounded results. Furthermore, MPFR/GMP

has floating-point arithmetic that is compatible with the IEEE754 standard.

The MPFR/GMP library has been developed using ANSI C standard. This library must be compiled and linked in every supported OS and C programming environment in which it is used. Therefore, MPFR/GMP is not feasible for use by novice users. Hence, in 2004, we developed the mpfr_gexpr code (source code: mpfr_gexpr.c) that was modified "gexpr.c". This code is a simple formula parser provided with GMP.

The mpfr_gexpr code can calculate constant values of an user-defined formula; it includes some elementary and special functions with user-defined precision in bits. For example, to obtain the value of sin(cos(3)) with a precision of 80 bits, can enter the following command:

```
./mpfr_gexpr -prec 80 'sin(cos(3.0))'
```

The value of $-8.3602186153773053407717876e{-}1$ is obtained by standard output. By using mpfr_gexpr, novice users who are unable to set up an MPFR/GMP environment can easily execute several mathematical functions with arbitrary precision provided by MPFR/GMP. We have developed and made available "Try MPFR!" (`http://ex-cs.sist.ac.jp/~tkouya/try_mpfr.html`) as an HTML and CGI interface on the Internet free of cost. Although the number of Try MPFR! users is limited, the service has never been at risk. We will be independently maintaining Try MPFR! with MPFR/GMP for unknown users.

The first objective developing MPFRgraph is to upgrade Try MPFR! service so that it is capable of displaying tables and graphs of any user-defined one-variable function.

3 Implementation of Mathematical Functions on the Web

Nowadays, the complexity in developing Web sites has necessitated the distribution of the development of codes among developers. For example, one developer can develop the HTML and CSS codes, another programmer can develop the JavaScript codes, and a third developer can develop the PHP codes. Large and complex Web sites cannot be built by power of one person. Since complex Web sites are developed by multiple developers, university students are unable to understand all the components of the developed Web sites. Hence, they are unable to gain a comprehensive knowledge of Web programming.

The second objective of developing MPFRgraph is to provide a real-time experience of developing a complex Web site to understand its functioning.

3.1 Development Environment of MPFRgraph

MPFRgraph has been developed in the following OS and programming environment:

Web Server Apache 2.2.3

PHP PHP 5.1.6, GD 2.0.28

Graph Library PHPlot 5.2.0, Open Flash Chart 2

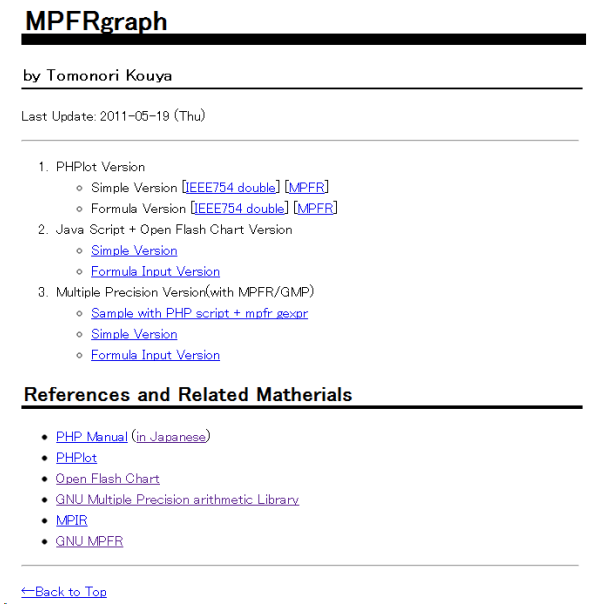


Figure 3: Educational materials related to MPFRgraph

Web Browser Google Chrome 12.0.742.53

MPFR/GMP MPFR 3.0.1, GMP 5.0.1

Web applications always execute with a combination of a Web server and a client (Web browser). On the Web server side, PHP codes must be implemented to execute the Web application. On the client side, JavaScript and Flash codes must be executed. In our recent version of MPFRgraph, arbitrary precision computation using MPFR/GMP is executed only on the Web server side because an arbitrary precision computation mechanism is currently absent in JavaScript and Flash.

However, the graph of an user-defined one-variable function can be plotted both on the Web server side and on the client side. Hence, MPFRgraph has two versions that differ in the mechanisms used to plot graphs. One version involves PHPlot running on the Web server side, whereas the other version involves Open Flash Chart running on the client side. We expect that both versions facilitate a good understanding the difference between them that produce the same results.

3.2 Process Flow of MPFRgraph

Figure 4 shows the detailed execution process of MPFRgraph. Processes **(1)** — **(4)** in Figure 4 are common for both the PHPlot and the Open Flash Chart versions. Processes **[5]** — **[7]** in Figure 4 correspond to the PHPlot version, and processes **(5)** — **(7)** correspond to the Open Flash Chart version.

The common processes in both versions are as follows:

(1) to access the HTML form to input the precision in bits, the mathematical expression of the user-defined one-variable function, the user-defined

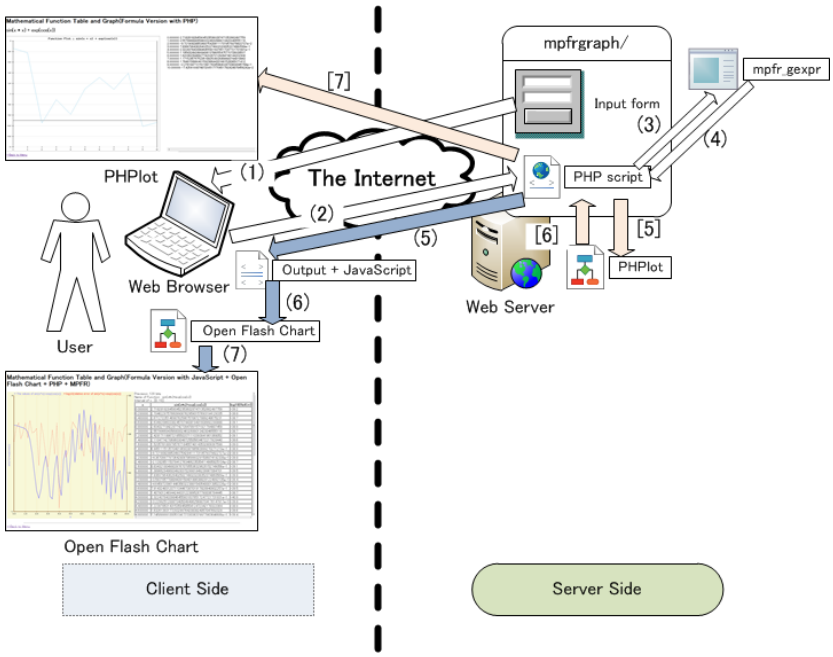


Figure 4: MPFRgraph Process Flow in detail

interval of the variable x, and the number of divisions in interval.

(2) to send the above inputs to the Web server by the GET method

(3) to transform the user-defined function to a constant expression (transform x to one set of values in the user-defined interval) in PHP script and send the transformed expression to the mpfr_gexpr code for execution

(4) to manipulate the result obtained by the mpfr_gexpr code to display its function table and to plot its graph.

The PHPlot version plots the graph on the Web server side ([**5**], [**6**]) and then sends both the function table and the PNG image of the graph to the Web browser on the client side ([**7**]). On the other hand, the Open Flash Chart version returns the values obtained the arbitrary precision to the client side ((**5**)) and then plots the graph using Flash Plug-in and JavaScript codes ((**6**), (**7**)) through the Web browser on the client side.

4 Courseware using MPFRgraph

We are preparing a programming textbook, entitled, "Workbook for Learning Web and HPC Programming (in Japanese)" (Figure 5) that would aid readers in developing Web and native applications in a command line interface (CLI) environment by remotely logging in to Linux. We will complete this textbook and involve MPFRgraph as one of educational materials in near future. The textbook contains the following chapters:

1. Linux as an operating system
2. Remote login to Linux
3. Manipulation of text files in a CLI
4. C programming and the first step to HPC programming
5. Learning Web programming with PHP
6. Three layers of Web programming with PHP and SQLite
7. Fusion of HPC and Web programming
8. Graph plotting on the Web with PHPlot
9. Mathematical function table built with Open Flash Chart or PHPlot (in progress).

Chapters 1 to 4 explain the technique for remoting logging into the Linux CLI. Chapter 5 provides an introduction to PHP programming. Chapter 6 deals with database programming with SQLite. Chapter 7 explains the technique for combining native applications with C and Web interfaces. Chapter 8 describes PHPlot used for plotting graphs on the Web. Chapter 9 combines all programming techniques discussed from chapters 1 to 8 and discusses the technique for developing MPFRgraph. Chapters 8 and 9 may be used as references to learn high-level Web programming.

This textbook was used for our lecture of Shizuoka Institute of Science and Technology in the first semester of 2011. An educational survey we conducted for this book reveals that very few students to understand the whole contents from chapter 1 to 7 because they were not familiar with Web programming before starting our lecture. However, they completed to build some Web sites explained in this textbook in detail.

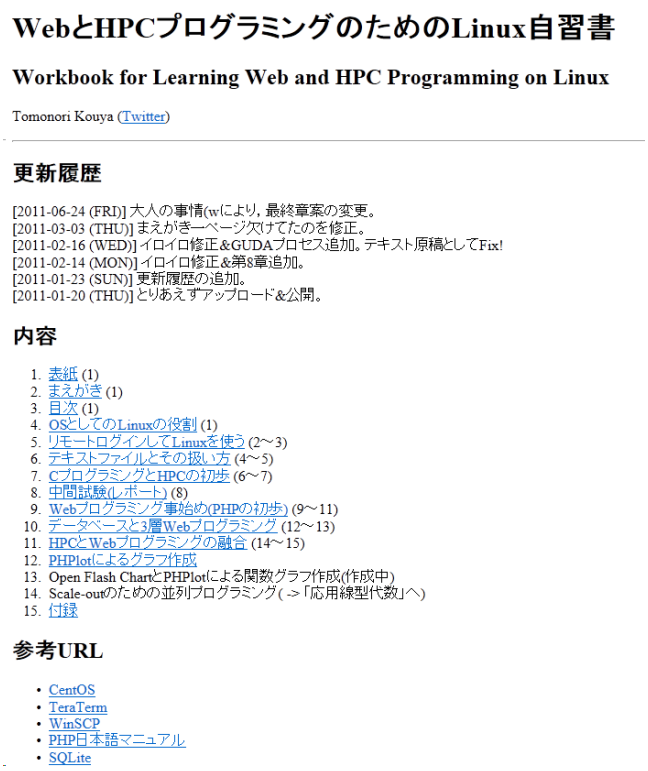
5 Conclusion and Future work

MPFRgraph has been developed an educational material for comprehensive learning the development of Web sites, and it has partially achieved success in attaining its goals. We will enhance this application and complete the textbook to further explain the development of MPFRgraph.

In the future, the following three enhancements will be made to MPFRgraph:

Adaptation to mainstream browsers The PHPlot version is compatible with mainstream Web browsers; however, the Open Flash Chart version is incompatible with on Internet Explorer 9. We aim to achieve compatibility of both versions with mainstream Web browsers and later with smart phones such as Android and iPhone, which have varying display sizes.

Save security It is necessary that MPFRgraph runs mpfr_gexpr on Web server side. But we currently have hesitation in saying that MPFRgraph is completely secure. Much of security check will be done.



WebとHPCプログラミングのためのLinux自習書

Workbook for Learning Web and HPC Programming on Linux

Tomonori Kouya (Twitter)

更新履歴

[2011-06-24 (FRI)] 大人の事情(wにより, 最終章案の変更。
[2011-03-03 (THU)] まえがき一ページ欠けてたのを修正。
[2011-02-16 (WED)] イロイロ修正&GUDAプロセス追加。テキスト原稿としてFix!
[2011-02-14 (MON)] イロイロ修正&第8章追加。
[2011-01-23 (SUN)] 更新履歴の追加。
[2011-01-20 (THU)] とりあえずアップロード&公開。

内容

参考URL

- CentOS
- TeraTerm
- WinSCP
- PHP日本語マニュアル
- SQLite

Figure 5: Educational materials related to MPFRgraph

As an application as approximation tool of any function required by users MPFRgraph dose not have any new mathematical ability, but it can be used as an interface to demonstrate the outcome of arbitrary precision computation. For example, it may be interesting to obtain interactive approximation of user-defined function at a user-defined precision level.

References

[1] Swox AB. The GNU Mmultiple Precision Arithmetic Library. `http://gmplib.org/`.

[2] L. J. Bayuk, Miguel de Benito, and Afan Ottenheimer. PHPlot. `http://phplot.sourceforge.net/`.

[3] John Glazebrook. Open Flash Chart 2. `http://teethgrinder.co.uk/open-flash-chart-2/`.

[4] Wolfram Research, Inc. Wolfram|Alpha. `http://www.wolframalpha.com/`.

[5] Daniel W. Lozier. NIST digital library of mathematical functions. *Annals of Mathematics and Artificial Intelligence*, Vol. 38, pp. 105–119, 2003.

[6] Milton Abramowitz and Irene A Stegun. *Handbook of Mathematical Functions*. Dover, 1964.

[7] Frank W. Olver, Daniel W. Lozier, Ronald F. Boisvert, and Charles W. Clark. *NIST Handbook of Mathematical Functions*. Cambridge University Press, New York, NY, USA, 1st edition, 2010.

[8] MPFR Project. The MPFR Library. `http://www.mpfr.org/`.

[9] Casio Computer Co., Ltd. $\sqrt{\text{keis}\alpha\text{n}}$. `http://keisan.casio.jp/`.

Tomonori Kouya, Ph.D. is interested in multiple precision numerical computation and its application to large-scale scienfic computation.
Postal Address: Department of Computer Science, Faculty of Comprehensive Informatics, Shizuoka Institute of Science and Technology,
2200-2 Toyosawa, Fukuroi 437-8555 Japan
E-mail: `tkouya@cs.sist.ac.jp`
Web site: `http://na-inet.jp/`